Knudsen compressor as a micro- and macroscale vacuum pump without moving parts or fluids

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Applications of Knudsen compressors as both microscale and macroscale vacuum pumps have been investigated. The study is based on a cascade analysis incorporating available transitional thermal transpiration and Poiseuille flow results for slender channels. It was found that the Knudsen compressor is an attractive possibility for microscale pumps down to a pressure of about 1 mTorr and for macroscale pumps to about 0.1 mTorr. A microscale pump for a micromass spectrometer providing a molecule flow rate of $5 \times 10^{14}$ molecules/s results in the following pump characteristics: energy use of 2.4 W, pump volume of 13.9 m$^3$/s at an inlet pressure of 1 mTorr and an energy use of 28.5 mW, and pump volume of 0.16 m$^3$/s at an inlet pressure of 10 mTorr. A macroscale pump providing a pumping speed of $10^3$ m$^3$/s results in a pump with an energy use of 1786 W, and pump volume of 1695 m$^3$/s at an inlet pressure of 0.1 mTorr. Several Knudsen compressor characteristics such as pressure rise, pumping speed, volume, energy use and mass flow are presented. © 1999 American Vacuum Society. [S0734-2101(99)05904-7]

I. INTRODUCTION

Investigating the phenomenon of thermal transpiration or thermal creep as the basis of a vacuum pump has had a sporadic history since the early 1900s. The phenomenon was first explained by Reynolds2,3 and contemporaneously investigated theoretically by Maxwell. In the early 1900s Knudsen2,3 studied a series of differentially heated and cooled capillaries to produce a staged compressor. As reported by Ebert, Gaede described a "thermopump" in unpublished notes during the same period. More recently several thermal transpiration pump configurations have been suggested.5–8 The driving reason for these attempts has been the attraction of a vacuum pump with no fluids and no moving components. The major disadvantages are that such pumps have low volume flow rates and tend to be inefficient in their use of energy. A further drawback is thought to be a low pressure limit for effective pumping significantly greater than 1 mTorr. The limit appears because it is generally assumed that certain critical components of the pump must be at least a factor of ten larger than the molecular mean free path in the process gas. Several related pump configurations, usually described as accommodation pumps, are based on the characteristic that some surfaces impacted by relatively cool molecules exhibit preferential directions for reflection. Accommodation pumps have been suggested6–11 to overcome the supposed low pressure limits of thermal transpiration pumping. Pham-Van-Diep et al.12 have described pumps or compressors that combine a modern version of Reynolds’ single porous thermal diffusion stage with Knudsen’s multi-stage compressor into a device called a Knudsen compressor. Subsequently several studies13–15 have demonstrated quantitatively the operation of a Knudsen compressor with up to three stages. A related device has been suggested8 relying on an interesting arrangement of temperature gradients along the interior surfaces in a channel. A proposed micromechanical thermal transpiration pump has very recently been described by Young.16

One aspect of thermal transpiration pumping that has been lacking is a realistic analysis of pumping performance and the associated energy use of the proposed devices. Turner7 presents a pumping speed analysis of Baum’s pump configuration assuming free molecular (collisionless) flow and suggests a higher throughput variant. Pham-Van-Diep12 discussed both throughput or upflow and the minimization of energy use per unit of mass upflow, assuming free molecular flow in the capillary sections of a Knudsen compressor cascade. More recently Muntz et al.17 have developed a transitional flow description of a Knudsen compressor cascade’s upflow and energy use, including the minimization of energy use per unit upflow. This analysis is used in the present article as the basis for a study of Knudsen compressors as microscale and macroscale vacuum pumps.

II. KNUDSEN COMPRESSOR BACKGROUND

Pham-Van-Diep et al.12 first outlined a microelectromechanical systems (MEMS) based thermal transpiration Knudsen compressor. This device is a modern version of Knudsen’s thermal transpiration compressor.2,3 A Knudsen compressor generates large changes in pressure by utilizing a cascade of multiple stages. Each stage is composed of a capillary and connector section. A temperature increase across the capillaries results in a thermal transpiration driven pressure increase. The capillary section is followed by a connector section where the pressure is approximately constant.
performance for a wide range of operating conditions. The 
provides more realistic estimates of Knudsen compressor 
analysis has been incorporated in a cascade analysis which 
has been presented in Sec. III.

III. ANALYSIS OF A THERMAL TRANSPERSION VACUUM PUMP

The original analysis of a Knudsen compressor’s performance was based on the assumption of an ideal situation of free-molecule flow in the capillary section and continuum flow in the connector section of a compressor stage. While these conditions can be closely matched in laboratory compressors, it is expected in practice that both the capillary and connector sections of the compressor frequently will operate in the transitional flow regime. Recently, Muntz et al. have replaced the two simultaneous assumptions of free-molecule and continuum flow by permitting transitional flow to exist in a modeled compressor stage. The modeling is accomplished using the results for transitional flows reported by Sone et al. The improved stage analysis has been incorporated in a cascade analysis which provides more realistic estimates of Knudsen compressor performance for a wide range of operating conditions. The analysis provided in Muntz et al. for full transitional modeling of both the capillary and connector sections of a Knudsen compressor cascade is summarized in this section. It provides the basis for the numerical simulations of microscale and macroscale Knudsen compressor vacuum pumps included in Sec. VI.

In a given cascade it is assumed that the average temperature is $T_{AVG} = T_{AVG,i} = T_{AVG,C,i}$, where $i$ is the stage number (a reference to Fig. 1 will be useful in the following descriptions). The subscript $i$ also refers to the capillary section unless noted otherwise and the subscript $C$ refers to the connector section. It follows that the absolute temperature difference across the $i$th stage is $|\Delta T_i| = |\Delta T_{C,i}|$ and it is assumed that $p_{AVG,i} = p_{AVG,C,i}$, the average pressure in the capillary section of stage $i$. The resulting expression for the stage pressure ratio is

$$P_i = 1 + \frac{p_{AVG,i}}{(p_{i-1})_{EFF}} \left[ \frac{\Delta T_i}{T_{AVG}} \left( \frac{Q_{T,i}}{Q_{P,i}} - \frac{Q_{T,C,i}}{Q_{P,C,i}} \right) - 2 \dot{M}_{DES} \right]$$

where $(p_{i-1})_{EFF}$ is the exit pressure of the $(i-1)$th stage or the entering pressure to the $i$th stage and $Q_T$ and $Q_P$ are the thermal transpiration and Poiseuille flow coefficients of each section, respectively. These flow coefficients are functions of the Knudsen number (Kn) and have been calculated for a large range of Kns for cylindrical tubes by Sone et al. The Knudsen number can be found from $Kn = (kT_{AVG})/ (\sqrt{2} \Omega p_{AVG}L_i)$ with $k$ and $\Omega$ being Boltzmann’s constant and the collision cross section of the working gas, respectively. The dimensions $L_i$ and $L_C$ refer to the length and pore radius of the capillaries. The upper-case subscript designates the connector section and the lower-case subscript designates the capillary section, $F$ is the fractional open area and $A$ is the cross-sectional area of a section. $\dot{M}_{DES}$ is the nondimensional design mass flow derived from $M_{DES} = \dot{M}_{DES}(2p_{AVG,1}/[(2kT_{AVG}/m)^{-1/2}A_1]$.

The presure ratio $p_{AVG,i}/(p_{i-1})_{EFF}$ is

$$\frac{p_{AVG,i}}{(p_{i-1})_{EFF}} = 1 + \frac{1}{2} \frac{\Delta T_i}{T_{AVG}} \left( \frac{Q_{T,i}}{Q_{P,i}} - \frac{Q_{T,C,i}}{Q_{P,C,i}} \right) - 2 \dot{M}_{DES} A_1 F A_1 p_{AVG,1} \left( \frac{L_i}{L_C} \right)^1 \left( \frac{L_C}{L_i} \right)^1$$

and to a good approximation for efficient operating conditions

$$\frac{p_{AVG,1}}{p_{AVG,i}} = \left( \frac{i-1}{i} \right)^{P_i}$$

Here it is understood that for $i = 1$, $p_{AVG,1}/p_{AVG,i} = 1$.

The relationship between $P_i$ and $\dot{M}_{DES}$ in Eq. (1) can be used to define a maximum mass flow $\dot{M}_{MAX}$ for a cascade. Since the first stage of the cascade controls the mass flow for the entire cascade under all reasonable operating conditions ($P_i \geq 1$), setting $i = 1$ and $P_i = 1$ in Eq. (1) gives

$$\dot{M}_{MAX} = F_1 \left[ \frac{\Delta T_{1}}{T_{AVG}} \left( \frac{Q_{T,1}}{Q_{P,1}} - \frac{Q_{T,C,1}}{Q_{P,C,1}} \right) \right] / \left( \frac{L_1}{L_C} \right)^1 + \left( \frac{L_C}{L_1} \right)^1$$

For all applications of the Knudsen compressor $\dot{M}_{DES} = \beta \dot{M}_{MAX}$ with $\beta$ having a value between 0 and 1. Initial
simplified studies of energy use per unit mass flow indicate a
$\beta$ of about 0.5 is appropriate. The pressure ratio generated for a cascade of $N$ stages is

$$\frac{N}{\delta_N} = \prod_{i=1}^{N} P_i.$$ (5)

The energy consumption for a block of $N$ stages can be found directly from

$$\dot{Q}_N = \sum_{i=1}^{N} \left( \frac{\Delta T_i}{K_i (1 - F_i) A_1} \right),$$ (6)

where $K_i$ is the thermal conductivity of the capillary membrane material. Thermal losses through the membrane are assumed to dominate the energy use. The cascade’s volume can be found from

$$V_B = \sum_{i=1}^{N} (A_1 L_{x,i} + A_{C,i} L_{X,i}).$$ (7)

For application of the Knudsen compressor as a vacuum pump it is of interest to study its low pressure pumping speed. The volume flow rate through the first stage can be written as

$$\dot{V} = A_1 \left( \frac{F_1 \beta \frac{\Delta T_1}{T_{AVG}} \sqrt{\frac{k}{2m} T_{AVG}}}{L_{x_1}} \right) \left( \frac{Q_{T,1}}{Q_{P,1}} - \frac{Q_{T,C,1}}{Q_{P,C,1}} \right).$$ (8)

IV. MICROSCALE AND MACROSCALE VACUUM PUMPS

Thermal transpiration vacuum pump designs for microscale and macroscale pumps are quite different tasks at the low pressure or inlet side. In microscale pumps with inlet pressures less than $10^{-2}$ Torr a significant but not necessarily fatal reduction in performance must be tolerated in order to preserve the microscale nature of the pump. Also, to rival a typical macroscale vacuum pump care is required to keep a thermal transpiration pump at a reasonable volume.

From a conventional perspective the major issue affecting the design of a thermal transpiration vacuum pump is the flow in the connector section. It has commonly been accepted that the connector section must have dimensions significantly greater than the mean free path of the process gas. For the first stages in the pump cascade this sets a minimum size for the connector that becomes quite large below $p_0 = 1$ mTorr, and at some low pressure undoubtedly makes thermal transpiration pumps too large. From the analysis reviewed in Sec. III it is interesting to examine the volume flow of the first stage of a Knudsen compressor. Using Eq. (8) plots of volume flow, normalized by the stage area $A_1$ and the square bracket grouping in the numerator of Eq. (8) (noted as $C_3$) for a range of $Kn_i$ (capillary Knudsen number) are shown in Fig. 2. In Fig. 2 the ratio of $L_R/L_r$ is used as a parameter to generate the separate pumping curves. Note that while there is a significant drop off in performance at high $Kn_i$, there is still pumping performance available for conditions that violate the accepted conditions of the mean free path in the connector section being comfortably less than the dimension $L_R$. For example, the connector flow for a capillary Knudsen number $Kn_i = 10^3$ and $L_R/L_r = 50$ (the uppermost curve in Fig. 2) has a Knudsen number $Kn_c = 20$.

The curves in Fig. 2 are a result of the shapes of the $Q_P$ and $Q_T$ flow coefficients as a function of $Kn$ for cylindrical tubes. These coefficients are shown in Fig. 3 for a large Knudsen number range. The significance of the ratio $L_R/L_r$ (indicating approximately $Kn_i/Kn_c$) is illustrated here by the horizontal range indications on the logarithmic Kn scale. Even at $Kn_i = 10^3$, it is evident that a $L_R/L_r = 50$ size difference leads to a small but useful difference in $Q_T$ (Fig. 3) and $Q_T/Q_P$. On the other hand a $L_R/L_r = 5$ gives a finite but much smaller difference in both $Q_T$ and $Q_T/Q_P$ (not shown).

The results in Fig. 2 are put in more concrete form in Fig. 4, where the volume flow per unit of the product $A_1 C_1$ is...
shown as a function of the average pressure in the first stage \((p_{AVG,1})\) for a range of capillary radii using the \(L_R/L_r=50\) case. Certainly, the Knudsen compressor requires a tradeoff between cascade size and minimum useful pumping pressure. However, as will be reported below compromises appear possible for both microscale and macroscale pumps. The investigation of the complete range of possibilities has only begun, the results presented here in Figs. 2 and 4 and others discussed later should only be treated as a very sparse indication of the possibilities. The procedures for optimizing the design of a pump cascade have not yet been completely developed, although they are under active investigation and will be reported in a later publication.

V. NUMERICAL VALIDATIONS

The previous analysis has been incorporated into a numerical code which provides a convenient method for comparing the performance of various Knudsen compressor geometries and input parameters. This section outlines the validation of the code using comparisons of numerical predictions to experimentally obtained Knudsen compressor data. The experimental data were provided from previous experiments using single-stage\(^1\) and multiple-stage\(^2\) configurations. The single-stage data were also used as the basis for comparisons between experimental results and transitional flow analysis predictions by Vargo \textit{et al.}\(^3\) These transitional flow studies provided useful insight into the capillary membrane’s physical properties such as pore size and open area. The experimental Knudsen compressor characteristics can be summarized as follows: capillary: \(L_r=0.225\ \mu m\), \(L_x=152\ \mu m\) and \(L_x/L_r=675.6\), connector: \(L_R=10\ 795\ \mu m\), \(L_X=76\ 200\ \mu m\) and \(L_X/L_R=7.06\). The experiments had a \(T_{AVG}=282\ K\) and a \(\Delta T/T_{AVG}\) of 0.084, 0.091 and 0.042 for the helium single-stage, nitrogen single-stage and helium two-stage configurations, respectively.

For both single- and multiple-stage experimental Knudsen

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**Fig. 3.** \(Q_T\) and \(Q_P\) coefficients for a range of Knudsen numbers.

**Fig. 4.** Normalized volume flow rate for a range of pressures.
compressor comparisons can be made to the experimental maximum pressure difference results. The maximum pressure difference in a compressor is the no upflow (M = 0) case and is found by noting the difference between the input and output pressures of the compressor at a given pressure. Numerical predictions are obtained by setting β = 0 which implies MDES = 0 in Eq. (1). Using Eq. (1) to find P1, the maximum pressure difference across a given stage is then \( \Delta p_{\text{MAX}} = p_{1,\text{EFF}} - (p_{i-1})_{\text{EFF}}. \) Maximum pressure difference curves for the one- and two-stage configurations were generated over a pressure range of 10–760 Torr. Comparisons of the numerical results and experiments are outlined in Fig. 5 for single-stage and two-stage Knudsen compressors. The numerical predictions using the transitional flow code provide reasonable agreement with the experimental data.

VI. MICROSCALE AND MACROSCALE VACUUM PUMP EXAMPLES

The validated numerical code utilizing the transitional flow analysis summarized in Sec. III can be used to provide estimates of Knudsen compressor performance for a range of inputs and cascade settings. In this section the results for a limited number of microscale and macroscale configurations for Knudsen compressor vacuum pump applications are presented. The estimates for a vacuum pump in each case depend on the values assigned to the many parameters needed to define a cascade. Thus, the results only give an indication of expected performance for the selected pumping tasks. The formidable job of defining optimum pump configurations subject to a variety of practical and theoretical constraints is currently being addressed. The pumps are presented here for comparison to each other as well as to conventional vacuum pumps.

One microscale and one macroscale device are identified to pump from an initial low pressure \( p_0 \) to atmospheric pressure. Both are assumed to be pumping nitrogen with the following inputs constant per stage and pump: \( T_{\text{AVG}} = 293 \, \text{K}, \) \( \Delta T = 100 \, \text{K}, \) \( F_i = 0.6, \) \( F_{C,i} = 1.0 \) and \( \beta = 0.5. \) Each pump is segmented into two sections with the first or low pressure section pumping from \( p_0 \) to a cutoff pressure and the second or high pressure section providing the remaining pumping needed to reach 1 atm. This is done primarily because the pump volume constraints are quite different for the microscale and macroscale pumps. In the simulations the low pressure section has the dimension \( L_{r,1} \) set to an initial value (for the micro case: \( L_{r,1} = 500 \, \mu\text{m} \) and \( L_R/L_A = 5; \) for the macro case: \( L_{r,1} = 5 \, \text{mm} \) and \( L_R/L_A = 20. \) In both cases \( L_{r,1} \) decreases as the cascade pressure increases in order to maintain a constant \( \text{Kn}_i. \) The first stage ends when the cascade pressure reaches a predetermined cutoff pressure (10 mTorr for the micro case and 50 mTorr for the macro case). At this point the second section of the Knudsen compressor performs the remaining pumping to atmosphere. In the second section the \( L_{r,1} \) is set to the first section’s initial value and \( L_{r,1} \) once again decreases under constant \( \text{Kn}_i \) (in this section \( \text{Kn}_i \) is much lower) until the pressure reaches 1 atm. For these examples, each stage of the pump has constant values of \( L_{C,i}, L_{R,i}, L_{X,i} \) and \( A_i = A_{r,i} \) which equal their first stage values for each section of the pump. Table I provides a summary of the microscale and macroscale vacuum pump examples along with several important measures of pump performance: the energy use per molecule of upflow \( Ql/N, \) the

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<th>( p_0 ) (mTorr)</th>
<th>( \dot{Q} ) (W)</th>
<th>( V ) (m/s)</th>
<th>( \dot{V} ) (mol/s)</th>
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cascade volume per molecule of upflow $V/N$, and the pumping speed per molecule of upflow $V/N$. Note that the macro and micropumps differ primarily in the initial $L_{r,1}$ and the $L_{R,1}$ values used as well as the pressure of the low pressure section to high pressure section switch (50 mTorr for the micro case and 10 mTorr for the macro case).

For the microscale pump assume a typical micromass spectrometer molecule flow rate of $5 \times 10^{14}$ molecules/s. The resulting important cascade characteristics are: for $p_0=1$ mTorr, $\dot{Q}=2.4$ W and $V=13.9$ m/$s$; and for $P_0=10$ mTorr, $\dot{Q}=28.5$ mW and $V=0.16$ m/$s$.

For a macroscale Knudsen compressor vacuum pump with a pumping speed of $10^3$ m/$s$ the cascade characteristics for $p_0=0.1$ mTorr are: $\dot{Q}=1786$ W and $V=1695$ m/$s$. For this pumping speed the energy consumption $\dot{Q}$ is very reasonable but the cascade volume is larger than would be convenient. A typical baffled diffusion pump in the 5000–10000 m/$s$ range might have as a figure of merit an energy consumption of 1.5 W/(m/$s$), whereas the Knudsen compressor pump in Table I requires 1.8 W/(m/$s$) at $p_0=0.1$ mTorr and 0.7 W/(m/$s$) at 1 mTorr.

### VII. SUMMARY

Applications of Knudsen compressors as both microscale and macroscale vacuum pumps have been investigated. The study is based on a cascade analysis incorporating available transitional thermal transpiration or creep flow results for slender channels. The slender channel presumption breaks down for the early, low pressure segment of the cascades so that the results should be treated as only initial investigations of the applications. Also, only a few ad hoc selections of discretionary cascade parameters have been used, with no systematic attempts at optimization for minimum energy use, minimum cascade volume or maximum pumping speed.

It was found that the Knudsen compressor is an attractive possibility for microscale pumps down to a pressure of about 1 mTorr and for macroscale pumps to about 0.1 mTorr. The attractiveness is attributed to the unique characteristics of Knudsen compressors such as low power, moderate size, no moving parts and no working fluid vacuum pumps. Another interesting observation is that the shape of the transitional flow coefficient curves as a function of $Kn$ offer the possibility of much lower pressure pumping than had been considered feasible for pure thermal transpiration pumps.